Chapter 1

Introduction

1.1 The Nature of Turbulent Flows

There are many opportunities to observe turbulent flows in our everyday surroundings, whether it be smoke from a chimney, water in a river or waterfall, or the buffeting of a strong wind. In observing a waterfall, we immediately see that the flow is unsteady, irregular, seemingly random and chaotic, and surely the motion of every eddy or droplet is unpredictable. In the plume formed by a solid rocket motor (see Fig. 1.1), turbulent motions of many scales can be observed, from eddies and bulges comparable in size to the width of the plume, to the smallest scales the camera can resolve. The features mentioned in these two examples are common to all turbulent flows.

More detailed and careful observations can be made in laboratory experiments. Figure 1.2 shows planar images of a turbulent jet at two different Reynolds numbers. Again, the concentration fields are irregular, and a large range of length scales can be observed.

As implied by the above discussion, an essential feature of turbulent flows is that the fluid velocity field varies significantly and irregularly in both position and time. The velocity field (which is properly introduced in Section 2.1) is denoted by $\mathbf{U}(\mathbf{x}, t)$, where $\mathbf{x}$ is the position and t is time.

Figure 1.3 shows the time history $U_1(t)$ of the axial component of velocity measured on the centerline of a turbulent jet (similar to that shown on Fig. 1.2). The horizontal line (in Fig. 1.3) shows the mean velocity denoted by $\langle U_1 \rangle$, and defined in Section 3.2. It may be observed that the velocity $U_1(t)$ displays significant fluctuations (about 25% of $\langle U_1 \rangle$), and that, far from being periodic, the time history shows variations on a wide range of

Figure 1.1: Photograph of the turbulent plume from the ground test of a Titan IV rocket motor. The nozzle exit diameter is 3m., the estimated plume height is 1,500m., and the estimated Reynolds number is 200×10^6. For more details see Mungal and Hollingsworth (1989). Permission to be obtained.

timescales. Very importantly, we observe that $U_1(t)$ and its mean $\langle U_1 \rangle$ are in some sense "stable": huge variations in $U_1(t)$ are not observed, nor does $U_1(t)$ spend long periods of time near values different from $\langle U_1 \rangle$.

Figure 1.4 shows the profile of the mean velocity $\langle U_1 \rangle$ measured in a similar turbulent jet as a function of the cross-stream coordinate x_2. In marked contrast to the velocity U_1, the mean velocity $\langle U_1 \rangle$ has a smooth profile, with no fine structure. Indeed, the shape of the profile is little different from that of a laminar jet.

In engineering applications turbulent flows are prevalent, but less easily seen. In the processing of liquids or gases with pumps, compressors, pipe lines, etc., the flows are generally turbulent. Similarly the flows around vehicles – e.g., airplanes, automobiles, ships and submarines – are turbulent. The mixing of fuel and air in engines, boilers, and furnaces, and the mixing

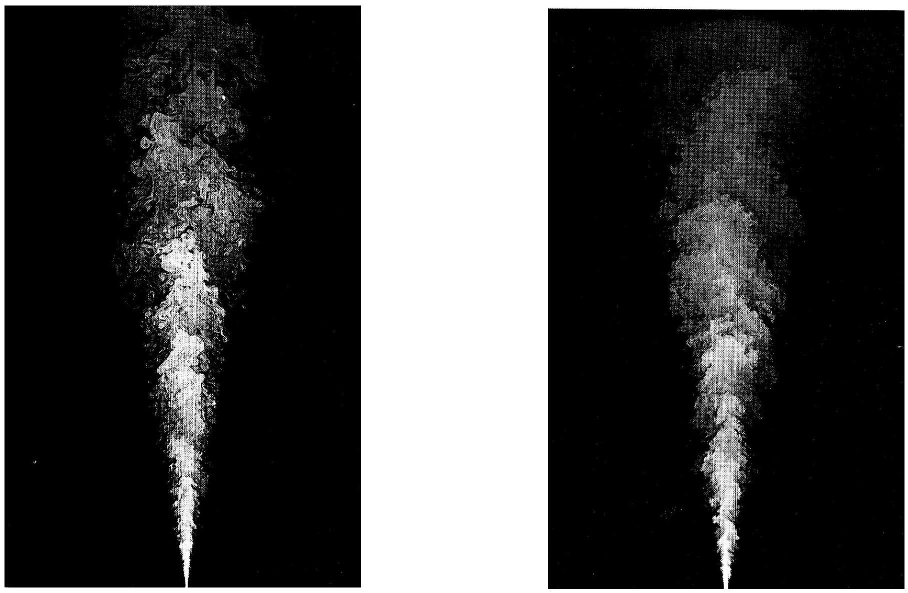

Figure 1.2: Planar images of concentration in a turbulent jet (a) Re = 5,000 (b) Re = 20,000. From Dahm and Dimotakis (1990) .

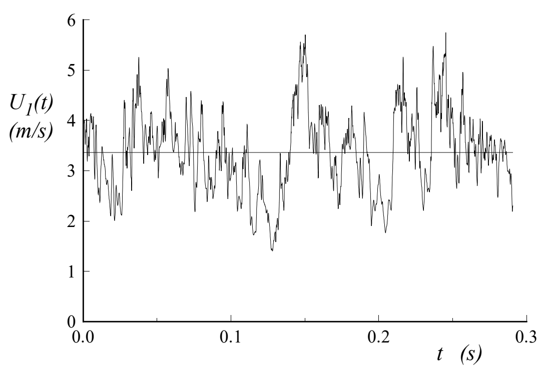

Figure 1.3: Time history of the axial component of velocity $U_1(t)$ on the centerline of a turbulent jet. From the experiment of Tong and Warhaft (1995).

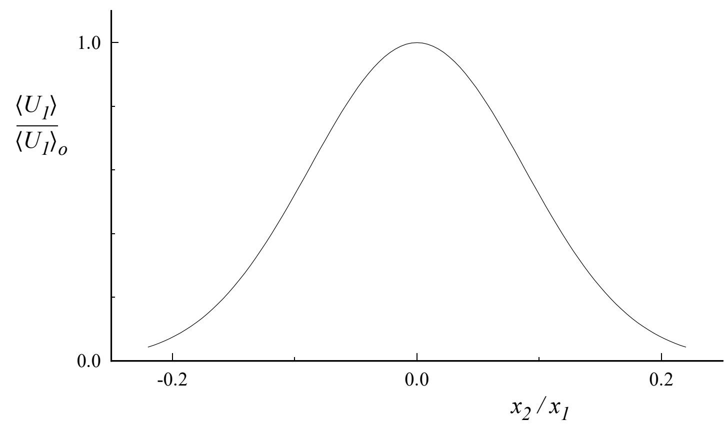

Figure 1.4: Mean axial velocity profile in a turbulent jet. The mean velocity $\langle U_1 \rangle$ is normalized by its value on the centerline, $\langle U_1 \rangle_0$; and the cross-stream (radial) coordinate x_2 is normalized by the distance from the nozzle x_1. The Reynolds number is 95,500. Adapted from Hussein, Capp, and George (1994).

of the reactants in chemical reactors take place in turbulent flows.

An important characteristic of turbulence is its ability to transport and mix fluid much more effectively than a comparable laminar flow. This is well demonstrated by an experiment first reported by Osborne Reynolds (1883). Dye is steadily injected on the centerline of a long pipe in which water is flowing. As Reynolds (1894) later established, this flow is characterized by a single non-dimensional parameter, now known as the Reynolds number Re. In general, it is defined by $\text{Re} = \mathcal{U}\mathcal{L}/\nu$, where $\mathcal{U}$ and $\mathcal{L}$ are characteristic velocity and length scales of the flow, and ν is the kinematic viscosity of the fluid. (For pipe flow, $\mathcal{U}$ and $\mathcal{L}$ are taken to be the area-averaged axial velocity and the pipe diameter, respectively.) In Reynolds' pipe-flow experiment if Re is less than about 2,300, the flow is laminar—the fluid velocity does not change with time, and all streamlines are parallel to the axis of the pipe. In this (laminar) case, the dye injected on the centerline forms a long streak which increases in diameter only slightly with downstream distances. If, on the other hand, Re exceeds about 4,000, then the flow is turbulent.[1] Close to the injector, the dye streak is jiggled about by the turbulent motion; it becomes progressively less distinct with downstream distance; and eventually mixing with the surrounding water reduces the peak dye concentration

[1] As the Reynolds number is increased, the transition from laminar to turbulent flow occurs over a range of Re, and this range depends on the details of the experiment.

to the extend that it is no longer visible.

(Visualizations from a reproduction of Reynolds' experiment, and from other canonical turbulent flows, are contained in Van Dyke 1982: there is great educational value in studying this collection of photographs.)

The effectiveness of turbulence to transport and mix fluids is of prime importance in many applications. When different fluid streams are brought together to mix , it is generally desirable for this mixing to take place as rapidly as possible. This is certainly the case for pollutant streams released into the atmosphere or into bodies of water, and for the mixing of different reactants in combustion devices and chemical reactors.

Turbulence is also effective at "mixing" the momentum of the fluid. As a consequence, on aircraft wings and ships' hulls the wall shear stress (and hence the drag) is much larger than it would be if the flow were laminar. Similarly, compared to laminar flow, heat and mass transfer rates at solid-fluid and liquid-gas interfaces are much enhanced in turbulent flows.

The major motivation for the study of turbulent flows is the combination of the three preceding observations: the vast majority of flows are turbulent; the transport and mixing of matter, momentum and heat in flows is of great practical importance; and, turbulence greatly enhances the rates of these processes.

1.2 The Study of Turbulent Flows

Many different techniques have been used to address many different questions concerning turbulence and turbulent flows. The first step towards providing a categorization of these studies is to distinguish between small-scale turbulence and the large-scale motions in turbulent flows.

As is discussed in detail in Chapter 6, at high Reynolds number there is a separation of scales . The large-scale motions are strongly influenced by the geometry of the flow (i.e., by the boundary conditions), and they control the transport and mixing. The behavior of the small scale motions, on the other hand, is determined almost entirely by the rate at which they receive energy from the large scales, and by the viscosity. Hence these small-scale motions have a universal character, independent of the flow geometry. It is natural to ask: what are the characteristics of the small-scale motions? Can they be predicted from the equations governing fluid motion? These are questions of *Turbulence Theory* which are addressed in the books of Batchelor (1953), Monin and Yaglom (1975), Panchev (1971), Lesieur (1990), McComb (1990), and others, and that are touched on only slightly in this book (in Chapter

6).

The focus of this book is on *Turbulent Flows*, studies of which can be divided into three categories:

(i) Discovery: experimental (or simulation) studies aimed at providing qualitative or quantitative information about particular flows.

(ii) Modelling: theoretical (or modelling) studies, aimed at developing tractable mathematical models that can accurately predict properties of turbulent flows.

(iii) Control : studies (usually involving both experimental and theoretical components) aimed at manipulating or controlling the flow or the turbulence in a beneficial way—for example, changing the boundary geometry to enhance mixing; or using active control to reduce drag.

The remainder of Part I of this book is based primarily on studies in the first category, the objective being to develop in the reader an understanding for the important characteristics of simple turbulent flows, of the dominant physical processes, and how they are related to the equations of fluid motion. The description of turbulent flows contained in Part I is not comprehensive: additional material can be found in the books of Monin and Yaglom (1971), Townsend (1976), Hinze (1975), and Schlichting (1979).

For studies in the second category, that aim at developing tractable mathematical models, the word "tractable" is crucial. For fluid flows, be they laminar or turbulent, the governing laws are embodied in the Navier-Stokes equations, which have been known for over a century. (These equations are reviewed in Chapter 2.) Considering the diversity and complexity of fluid flows, it is quite remarkable that the relatively simple Navier-Stokes equations describe them accurately and in complete detail. But in the context of turbulent flows, their power is also their weakness: the equations describe every detail of the turbulent velocity field from the largest to the smallest length and time scales. The amount of information contained in the velocity field is vast, and as a consequence (in general) the direct approach of solving the Navier-Stokes equations is impossible. So, while the Navier-Stokes equations accurately describe turbulent flows, they do not provide a *tractable* model for them.

The direct approach of solving the Navier-Stokes equations for turbulent flows is called Direct Numerical Simulation (DNS), and is described in Chapter 9. While DNS is intractable for the high Reynolds number flows of practical interest, it is nevertheless a powerful research tool for investigating simple turbulent flows at moderate Reynolds numbers. In the description of wall-bounded flows in Chapter 7, DNS results are used extensively to explore the physical processes involved.

For the high Reynolds number flows that are prevalent in applications, the natural alternative is to pursue a statistical approach. That is, to describe the turbulent flow, not in terms of the velocity $\mathbf{U}(\mathbf{x}, t)$, but in terms of some statistics, the simplest being the mean velocity field $\langle \mathbf{U}(\mathbf{x}, t) \rangle$. A model based on such statistics can lead to a tractable set of equations, because statistical fields vary smoothly (if at all) in position and time. In Chapter 3 we present the concepts and techniques used in the statistical representation of turbulent flow fields; while in Part II we describe statistical models that can be used to calculate the properties of turbulent flows. The approaches described include: turbulent viscosity models, e.g., the k-ε model (Chapter 10); Reynolds-stress models (Chapter 11); models based on the probability density function (PDF) of velocity (Chapter 12); and Large Eddy Simulations (LES) (Chapter 13).

The statistical models described in Parts II can be used in some studies in the third category mentioned above – that is, studies aimed at manipulating or controlling the flow of the turbulence. But such studies are not explicitly discussed here.

A broad range of mathematical techniques is used to describe and model turbulent flows. Appendices on several of these techniques are provided to serve as brief tutorials and summaries. The first of these is on Cartesian tensors which are used extensively. The reader may wish to review this material (Appendix A) before proceeding. There are exercises throughout the book which provide the reader with the opportunity to practice the mathematical techniques employed. Most of these exercises also contain additional results and observations. A list of Nomenclature and Abbreviations is provided on page xvii.